P.G. Sem - II , CC - VII

Form an integral equation corresponding to the differential equation given by

$$\frac{d^3y}{dx^3} + x\frac{d^2y}{dx^2} + (x^2 - x)y = xe^x + 1$$

With initial conditions $y(0) = 1 = y'(0)$,

$$y''(0) = 0$$

Soln

The given differential equation is

$$\frac{d^3y}{dx^3} + x\frac{d^2y}{dx^2} + (x^2 - x)y = xe^x + 1 \quad \text{--- (1)}$$

with initial conditions

$$y(0) = 1 = y'(0),\ y''(0) = 0 \quad \text{--- (2)}$$

Let us suppose

$$\frac{d^3y}{dx^3} = u(x) \quad \text{--- (3)}$$

Integrating (3) b.s.w.r. to $x$ from $0$ to $x$ we get

$$\frac{d^2y}{dx^2} = \int_0^x u(t)\,dt \quad \text{--- (4)}$$

Again integrating w.r.to $x$ from $0$ to $x$ we get

$$\frac{dy}{dx} = \int_0^x (x-t)\,u(t)\,dt + 1$$

Similarly, we get

$$y = \frac{1}{\lfloor 2}\int_0^x (x-t)^2\,u(t)\,dt + x + 1 \quad \text{--- (5)}$$

Now, putting all these values in the given equation (1), we get

$$u(x) + x\left[\int_0^x u(t)\,dt\right] + (x^2 - x)\left[\frac{1}{2}\int_0^x (x-t)^2 u(t)\,dt + x + 1\right] = xe^x + 1$$

$$\Rightarrow u(x) = xe^x + 1 - x(x^2 - 1) - \int_0^x \left[x + \frac{1}{2}(x^2 - x)(x-t)^2\right] u(t)\,dt$$

Which is the required integral equation